M12/5/MATSD/SP2/ENG/TZ2/XX

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**MATHEMATICAL STUDIES**
**STANDARD LEVEL**
**PAPER 2**

Friday 4 May 2012 (morning)

1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematical Studies SL*** *information booklet* is required for this paper.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is *[90 marks]*.

*Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.*

**1.** *[Maximum mark: 21]*

Leanne goes fishing at her favourite pond. The pond contains four different types of fish: bream, flathead, whiting and salmon. The fish are either undersized or normal. This information is shown in the table below.

| Size / Type of fish | Bream | Flathead | Whiting | Salmon | Total |
|---|---|---|---|---|---|
| **Undersized** | 3 | 12 | 18 | 9 | 42 |
| **Normal** | 0 | 11 | 24 | 13 | 48 |
| **Total** | 3 | 23 | 42 | 22 | |

(a) Write down the total number of fish in the pond. *[1 mark]*

Leanne catches a fish.

(b) Find the probability that she

(i) catches an undersized bream;

(ii) catches either a flathead or an undersized fish or both;

(iii) does **not** catch an undersized whiting;

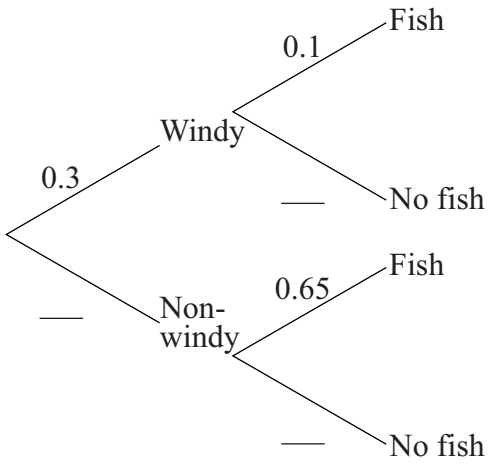
(iv) catches a whiting given that the fish was normal. *[7 marks]*

*(This question continues on the following page)*

*(Question 1 continued)*

Leanne notices that on windy days, the probability she catches a fish is $0.1$ while on non-windy days the probability she catches a fish is $0.65$. The probability that it will be windy on a particular day is $0.3$.

(c) **Copy and complete** the probability tree diagram below. *[3 marks]*




(d) Calculate the probability that it is windy and Leanne catches a fish on a particular day. *[2 marks]*

(e) Calculate the probability that Leanne catches a fish on a particular day. *[3 marks]*

(f) Use your answer to part (e) to calculate the probability that Leanne catches a fish on two consecutive days. *[2 marks]*

(g) Given that Leanne catches a fish on a particular day, calculate the probability that the day was windy. *[3 marks]*

**2.** *[Maximum mark: 14]*

Cedric wants to buy an €8000 car. The car salesman offers him a loan repayment option of a 25 % deposit followed by 12 equal monthly payments of €600.

(a) Write down the amount of the deposit. *[1 mark]*

(b) Calculate the total cost of the loan under this repayment scheme. *[2 marks]*

Cedric's mother decides to help him by giving him an interest free loan of €8000 to buy the car. She arranges for him to repay the loan by paying her €$x$ in the first month and €$y$ in every following month until the €8000 is repaid.

The total amount that Cedric's mother receives after **12** months is €3500.
This can be written using the equation $x+11y=3500$.
The total amount that Cedric's mother receives after **24** months is €7100.

(c) Write down a second equation involving $x$ and $y$. *[1 mark]*

(d) Write down the value of $x$ and the value of $y$. *[2 marks]*

(e) Calculate the number of months it will take Cedric's mother to receive the €8000. *[3 marks]*

Cedric decides to buy a cheaper car for €6000 and invests the remaining €2000 at his bank. The bank offers two investment options over three years.

Option A: Compound interest at an annual rate of 8 %.

Option B: Compound interest at a nominal annual rate of 7.5 %, **compounded monthly**.

***Express each answer in part (f) to the nearest euro.***

(f) Calculate the value of his investment at the end of three years if he chooses

(i) Option A;

(ii) Option B. *[5 marks]*

**3.** *[Maximum mark: 18]*

200 people were asked the amount of time $T$ (minutes) they had spent in the supermarket. The results are represented in the table below.

| Time ($T$) | $0 < T \leq 10$ | $10 < T \leq 20$ | $20 < T \leq 30$ | $30 < T \leq 40$ | $40 < T \leq 50$ |
|---|---|---|---|---|---|
| Number of people | 23 | 57 | 93 | 21 | 6 |

(a) State if the data is discrete or continuous. *[1 mark]*

(b) State the modal group. *[1 mark]*

(c) Write down the midpoint of the interval $10 < T \leq 20$. *[1 mark]*

(d) Use your graphic display calculator to find an estimate for

(i) the mean;

(ii) the standard deviation. *[3 marks]*

The results are represented in the cumulative frequency table below, with upper class boundaries of 10, 20, 30, 40, 50.

| Upper class boundaries | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|
| Cumulative frequency | 23 | 80 | 173 | $q$ | $r$ |

(e) Write down the value of

(i) $q$;

(ii) $r$. *[2 marks]*

(f) On graph paper, draw a cumulative frequency graph, using a scale of 2 cm to represent 10 minutes ($T$) on the horizontal axis and 1 cm to represent 10 people on the vertical axis. *[4 marks]*

(g) **Use your graph** from part (f) to estimate

(i) the median;

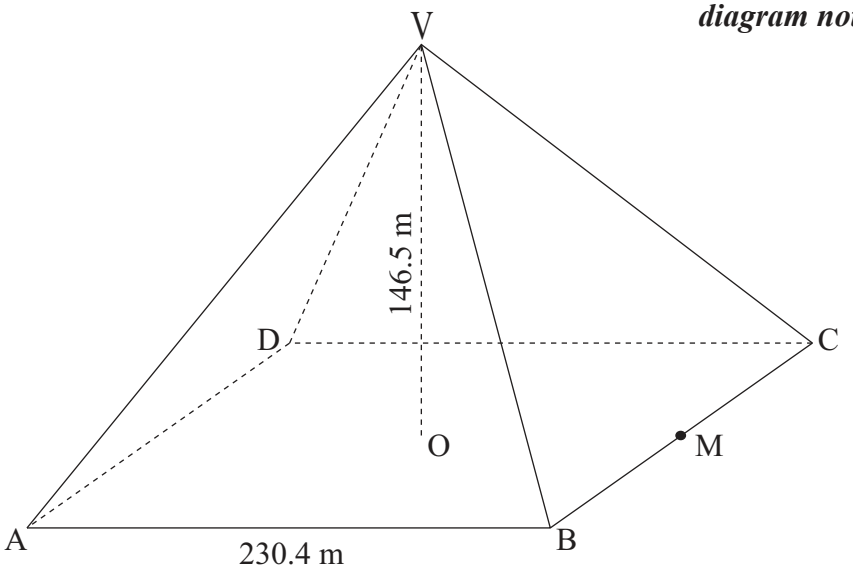
(ii) the 90$^{th}$ percentile of the results;

(iii) the number of people who shopped at the supermarket for more than 15 minutes. *[6 marks]*

**4.** *[Maximum mark: 18]*

The Great Pyramid of Cheops in Egypt is a square based pyramid. The base of the pyramid is a square of side length 230.4 m and the vertical height is 146.5 m. The Great Pyramid is represented in the diagram below as ABCDV. The vertex V is directly above the centre O of the base. M is the midpoint of BC.

***diagram not to scale***




(a) (i) Write down the length of OM.

(ii) Find the length of VM. *[3 marks]*

(b) Find the area of triangle VBC. *[2 marks]*

(c) Calculate the volume of the pyramid. *[2 marks]*

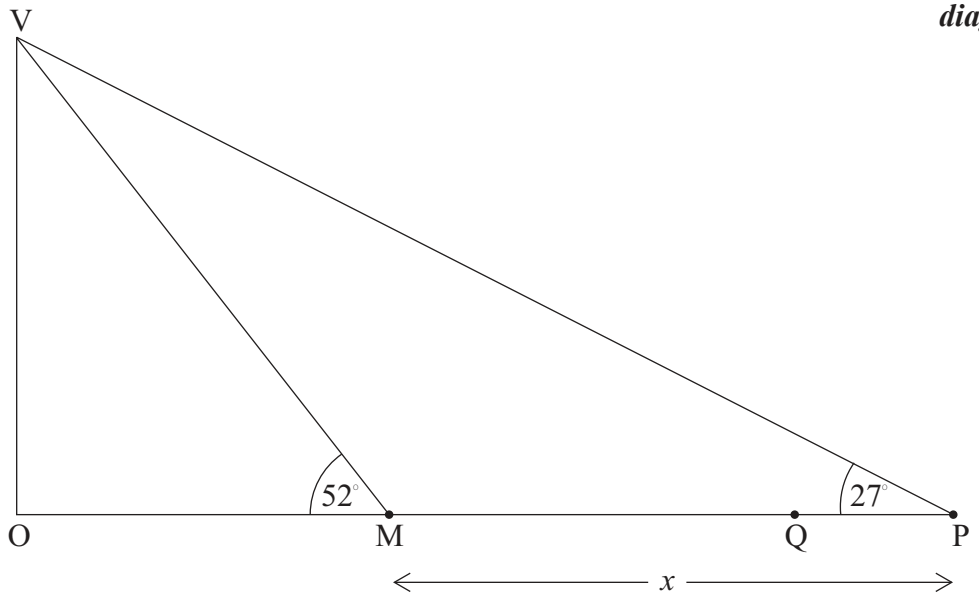
(d) Show that the angle between the line VM and the base of the pyramid is $52^\circ$ correct to 2 significant figures. *[2 marks]*

*(This question continues on the following page)*

*(Question 4 continued)*

Ahmed is at point P, a distance $x$ metres from M on horizontal ground, as shown in the following diagram. The size of angle VPM is $27^\circ$. Q is a point on MP.

***diagram not to scale***




(e) Write down the size of angle VMP. *[1 mark]*

(f) Using your value of VM from part (a)(ii), find the value of $x$. *[4 marks]*

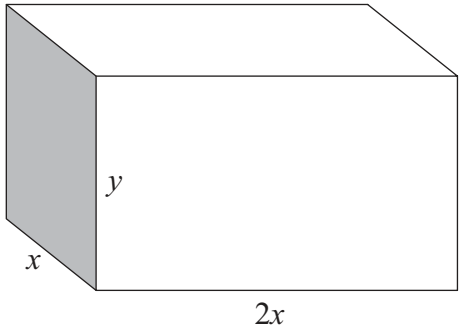
Ahmed walks 50 m from P to Q.

(g) Find the length of QV, the distance from Ahmed to the vertex of the pyramid. *[4 marks]*

**5.** *[Maximum mark: 19]*

A shipping container is to be made with six rectangular faces, as shown in the diagram.

***diagram not to scale***




The dimensions of the container are
length $2x$
width $x$
height $y$.

All of the measurements are in metres. The total length of all twelve edges is 48 metres.

(a) Show that $y = 12 - 3x$. *[3 marks]*

(b) Show that the volume $V$ m$^3$ of the container is given by

$$V = 24x^2 - 6x^3$$ *[2 marks]*

(c) Find $\frac{dV}{dx}$. *[2 marks]*

(d) Find the value of $x$ for which $V$ is a maximum. *[3 marks]*

(e) Find the maximum volume of the container. *[2 marks]*

(f) Find the length and height of the container for which the volume is a maximum. *[3 marks]*

The shipping container is to be painted. One litre of paint covers an area of 15 m$^2$. Paint comes in tins containing four litres.

(g) Calculate the number of tins required to paint the shipping container. *[4 marks]*